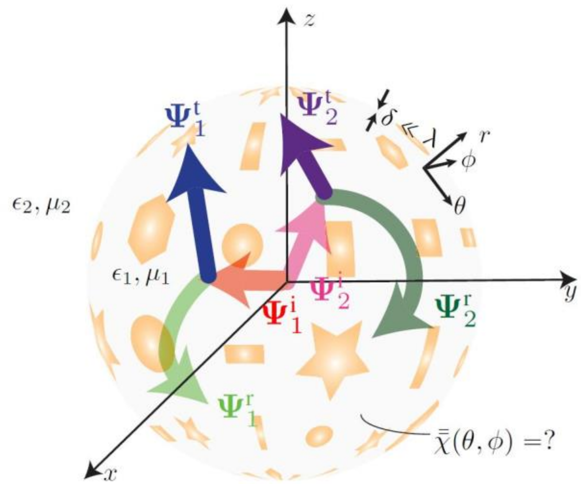
Spherical Metasurfaces

Proc. Title: Multiple Beam Forming using Spherical Metasurfaces

Xiao Jia, Yousef Vahabzedeh, Christophe Caloz, and Fan Yang



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Abstract

Spherical metasurfaces are particularly suited for the beam forming as they intrinsically cover the entire radiation space. We introduce here the possibility to realize multiple simultaneous and independent beam forming operations with a metasurface synthesized by Generalized Sheet Transition Conditions (GSTCs) combined with bianisotropic surface susceptibility tensors. Specifically, we demonstrate a spherical metasurface tilting upwards and downwards the radiation patterns of an electric dipole and a magnetic dipole, respectively.

Keywords: Spherical metasurface, Generalized Sheet Transition Conditions (GSTCs), bianisotropic surface susceptibility tensors, synthesis, radiation pattern control.

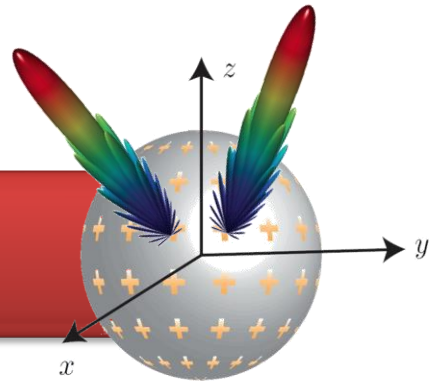
Biography

Xiao Jia received the B.S. degree from Northwestern Polytechnical University, Xi'an, China, the M.S. degree form University of Chinese Academy of Sciences, Beijing, China, in 2013 and 2016, respectively, She received the Ph.D. degree from Tsinghua University, Beijing, China, in 2020, under the supervision of Prof. Fan Yang. From 2018 to 2019, she was a visiting student at Polytechnique Montréal, Montréal, Québec, Canada, with the supervision of Prof. Christophe Caloz.

She is currently a lecturer at Beijing Jiaotong University, Beijing, China. Her current research interests include computational electromagnetics, metasurfaces, computational electromagnetics, reflectarray and transmitarray.

Outline

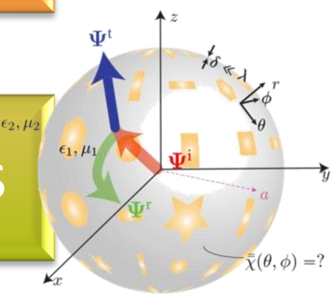
I. Background & Motivation



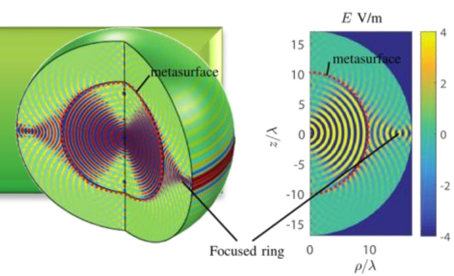
II. Boundary Conditions

$$\hat{\mathbf{r}} \times \triangle\mathbf{E} = -j\omega\mu_0\mathbf{M}_{\mathrm{s},\parallel} + \nabla_\parallel(P_{\mathrm{s},r}/\epsilon_0) \times \hat{\mathbf{r}} - K_{\parallel,\mathrm{s,imp}},$$
$$\hat{\mathbf{r}} \times \triangle\mathbf{H} = j\omega\mathbf{P}_{\mathrm{s},\parallel} - \hat{\mathbf{r}} \times \nabla_\parallel M_{\mathrm{s},r} + J_{\parallel,\mathrm{s,imp}},$$

III. Spherical Metasurface Synthesis



IV. Illustrative Examples



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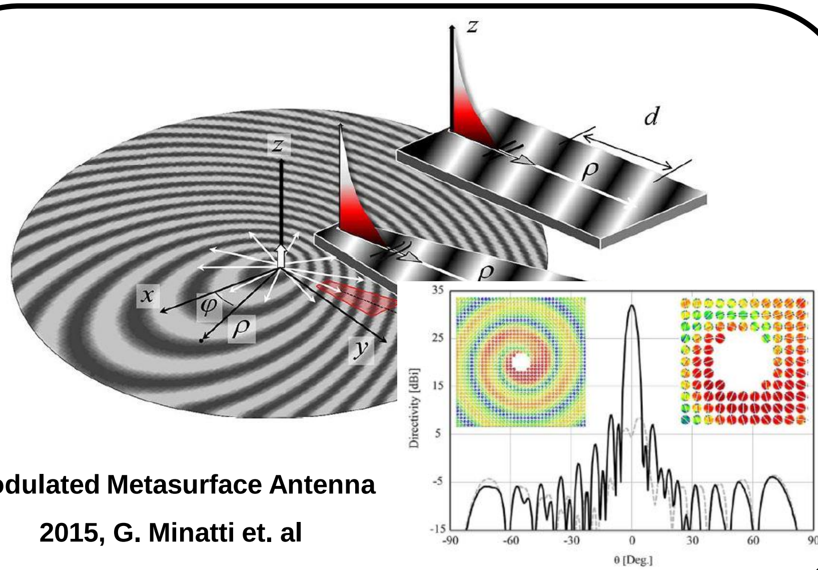
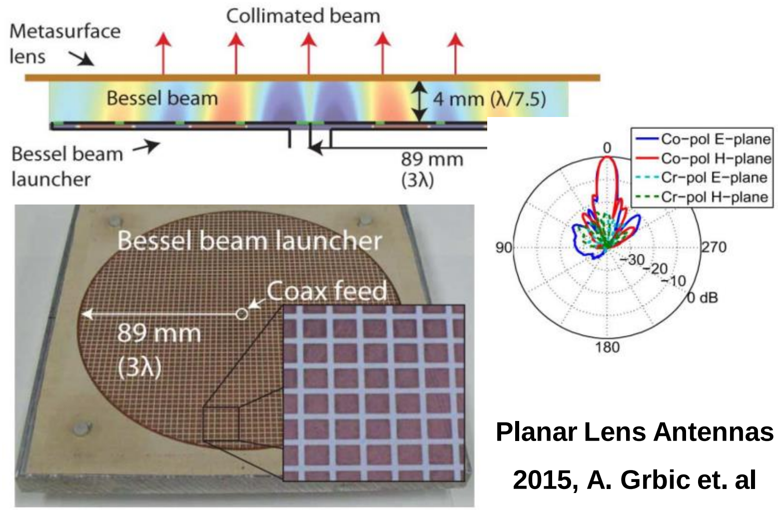
Beam Forming with Planar Metasurfaces



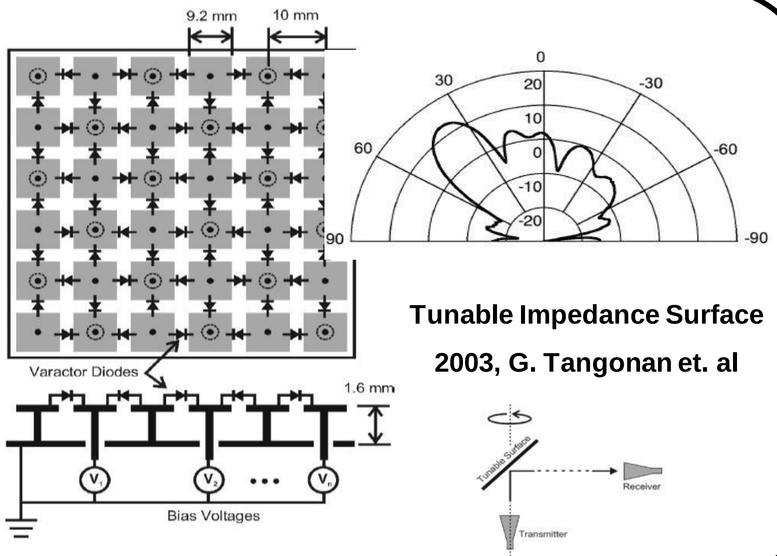
Modulated Metasurface Antenna

2015, G. Minatti et. al



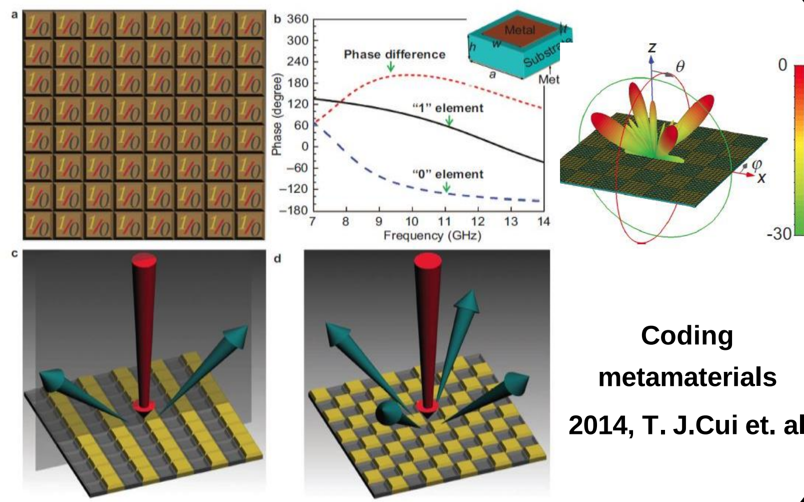
Planar Lens Antennas

2015, A. Grbic et. al



Tunable Impedance Surface

2003, G. Tangonan et. al



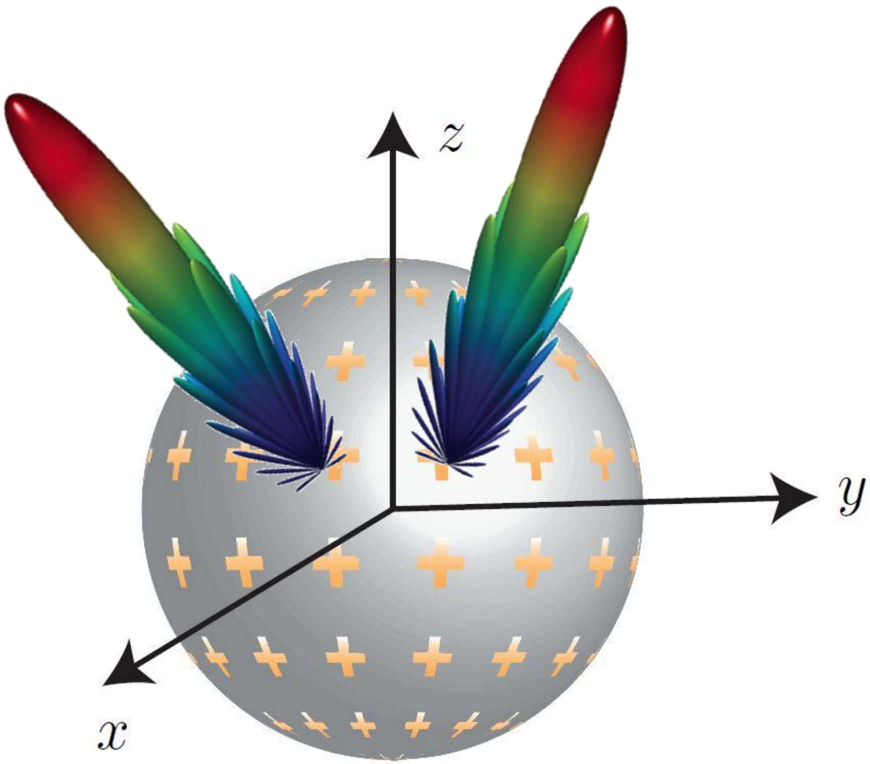
Coding metamaterials

2014, T. J.Cui et. al

Disadvantage: undesirable radiation in specific directions.

Features of Spherical Metasurfaces

Advantages: Radiation pattern control over entire 4π steradian solid angle (all 3D space) !



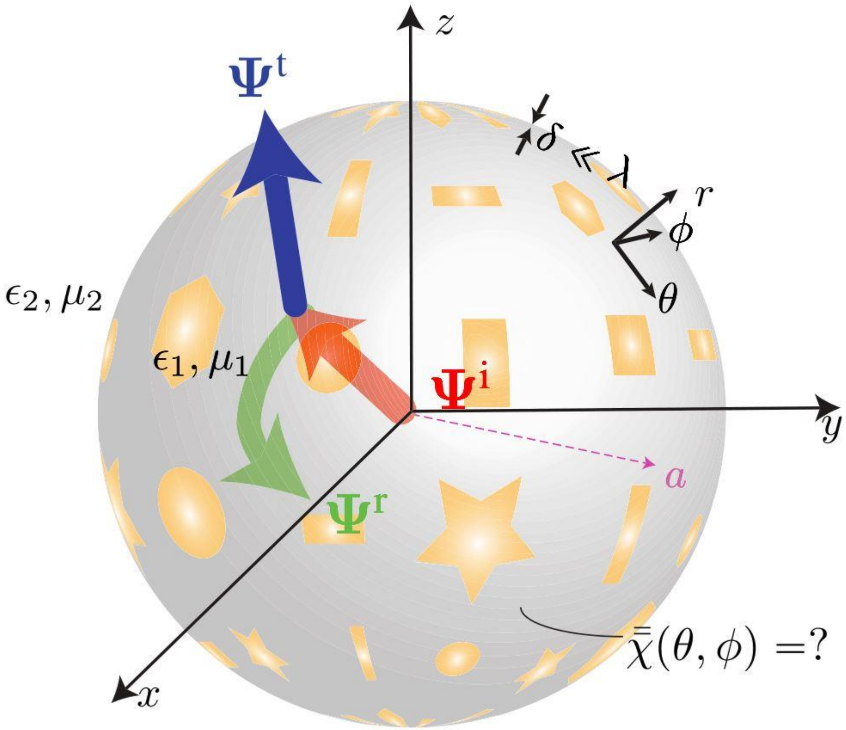
Features:

- Closed structures (without diffraction at edges)
- Decreasing size of particles from equator to poles
- Possible multiple reflection events (porous spherical cavity)

Assumptions: a) Interior excitation, b) zero reflection (matched)

Ref: Raeker, Brian O., and Scott M. Rudolph. "Arbitrary Transformation of Radiation Patterns Using a Spherical Impedance Metasurface." *IEEE Transactions on Antennas and Propagation* 64.12 (2016): 5243-5250.

Synthesis Problem of Spherical Metasurface



$$\begin{bmatrix} \chi_{\mathrm{ab}}^{rr}(\theta,\phi) & \chi_{\mathrm{ab}}^{r\theta}(\theta,\phi) & \chi_{\mathrm{ab}}^{r\phi}(\theta,\phi) \\ \chi_{\mathrm{ab}}^{\theta r}(\theta,\phi) & \chi_{\mathrm{ab}}^{\theta\theta}(\theta,\phi) & \chi_{\mathrm{ab}}^{\theta\phi}(\theta,\phi) \\ \chi_{\mathrm{ab}}^{\phi r}(\theta,\phi) & \chi_{\mathrm{ab}}^{\phi\theta}(\theta,\phi) & \chi_{\mathrm{ab}}^{\phi\phi}(\theta,\phi) \end{bmatrix}$$

$$(\mathrm{a,b}) = (\mathrm{e,e}),\ (\mathrm{e,m}),\ (\mathrm{m,e})\ \text{and}\ (\mathrm{m,m})$$

Synthesis problem:

- **Determine susceptibility tensors:**

$$E(\theta,\varphi), \mathrm{H}(\theta,\varphi) \rightarrow \bar{\bar{\chi}}(\theta,\varphi)$$

- **Determine scattering particles:**

$$\chi_{ee}^{\theta\theta} = \chi_{ee}^{\varphi\varphi} \rightarrow$$

$$\chi_{em}^{\theta r} = \chi_{em}^{\varphi r} \rightarrow$$

Ref: Jia, Xiao, et al. "A general synthesis method of spherical metasurfaces based on susceptibility tensors." IEEE Transactions on Antennas and Propagation. (under review)

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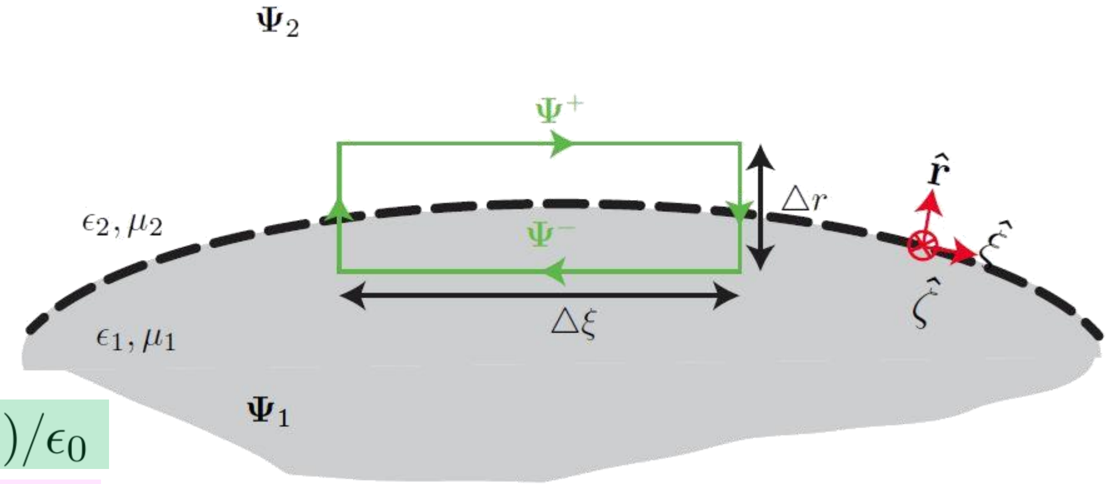
III. Spherical Metasurface Synthesis

IV. Illustrative Examples

Maxwell Equations in Terms of Volume & Surface Polarizations



Maxwell Equations:

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} - \mathbf{K}_{\text{imp}}$$
$$\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}_{\text{imp}}$$

The Constitutive Relations:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \quad \text{or} \quad \mathbf{E} = (\mathbf{D} - \mathbf{P})/\epsilon_0$$
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}), \quad \text{or} \quad \mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$$

Medium information in polarizibilities:

$$\nabla \times [(\mathbf{D} - \mathbf{P})/\epsilon_0)] = -j\omega \mu_0 (\mathbf{H} + \mathbf{M}) - \mathbf{K}_{\text{imp}}$$
$$\nabla \times (\mathbf{B}/\mu_0 - \mathbf{M}) = j\omega (\epsilon_0 \mathbf{E} + \mathbf{P}) + \mathbf{J}_{\text{imp}}$$

$$\mathbf{P} \rightarrow C/m^2 \qquad \mathbf{M} \rightarrow A/m$$
$$\mathbf{P}_{\text{s}} \rightarrow C/m \qquad \mathbf{M}_{\text{s}} \rightarrow A$$

Polarizations decomposition

$$\mathbf{P} = \mathbf{P}_{\text{v}} + \mathbf{P}_{\text{s}} \delta(r) \qquad \mathbf{M} = \mathbf{M}_{\text{v}} + \mathbf{M}_{\text{s}} \delta(r)$$

$$\nabla \times \left(\frac{\mathbf{D} - \mathbf{P}_{\text{v}}}{\epsilon_0} \right) = -j\omega \mu_0 \mathbf{H} - j\omega \mu_0 \mathbf{M}_{\text{v}} - j\omega \mu_0 \mathbf{M}_{\text{s}} + \nabla \times \left[\frac{\mathbf{P}_{\text{s}} \delta(r)}{\epsilon_0} \right] - \mathbf{K}_{\text{imp}}$$

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}_{\text{v}} \right) = j\omega \epsilon_0 \mathbf{E} + j\omega \mathbf{P}_{\text{v}} + j\omega \mathbf{P}_{\text{s}} + \nabla \times [\mathbf{M}_{\text{s}} \delta(r)] + \mathbf{J}_{\text{imp}}$$

Maxwell Equations in Terms of Volume & Surface Polarizations

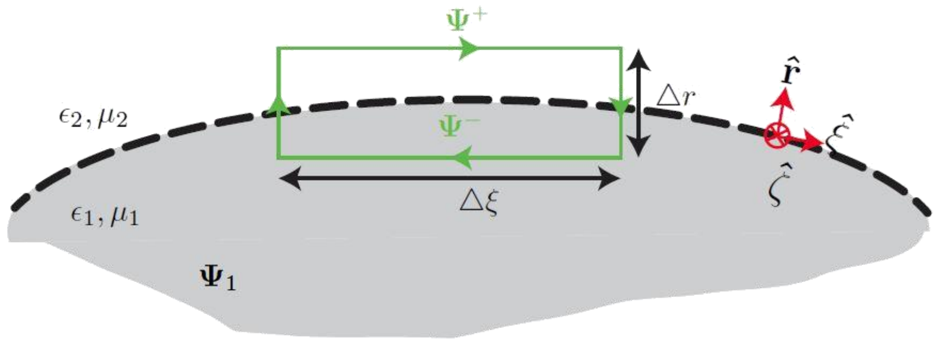
Applying Stokes Theorem

$$\oint\left(\frac{\mathbf{D}-\mathbf{P}_\text{v}}{\epsilon_0}\right)\cdot d\mathbf{l} = -j\omega\mu_0\iint(\mathbf{H}+\mathbf{M}_\text{v})\cdot d\mathbf{S} - j\omega\mu_0\iint\delta(r)\mathbf{M}_\text{s}\cdot d\mathbf{S} + \oint\left[\frac{\delta(r)\mathbf{P}_\text{s}}{\epsilon_0}\right]\cdot d\mathbf{l} - \iint\mathbf{K}_\text{imp}\cdot d\mathbf{S}$$

$$\oint\left(\frac{\mathbf{B}}{\mu_0}-\mathbf{M}_\text{v}\right)\cdot d\mathbf{l} = j\omega\epsilon_0\iint(\mathbf{E}+\mathbf{P}_\text{v}/\epsilon_0)\cdot d\mathbf{S} + j\omega\iint_{\mathbf{\Psi}_2}\delta(r)\mathbf{P}_\text{s}\cdot d\mathbf{S} + \oint[\mathbf{M}_\text{s}\delta(r)]\cdot d\mathbf{l} + \iint\mathbf{J}_\text{imp}\cdot d\mathbf{S}$$

with $\dfrac{\mathbf{D}^\pm-\mathbf{P}_\text{v}^\pm}{\epsilon_0} = \mathbf{E}^\pm$

yields



$$(E_\xi^+ - E_\xi^-)\triangle\xi + (-E_{r,\text{right}} + E_{r,\text{left}})\triangle r$$
$$= -j\omega\mu_0(H_\zeta + M_{\text{v},\zeta})\triangle\xi\triangle r - j\omega\mu_0 M_{\text{s},\zeta}\delta(r)\triangle\xi\triangle r - (P_{\text{s},r,\text{right}} - P_{\text{s},r,\text{left}})\delta(r)\triangle r/\epsilon_0$$
$$- K_{\zeta,\text{v,imp}}\triangle\xi\triangle r - K_{\zeta,\text{s,imp}}\delta(r)\triangle\xi\triangle r$$

$$\triangle r \to 0 \quad \downarrow \quad \delta(r)\triangle r \to 1$$

$$(E_\xi^+ - E_\xi^-) = -j\omega\mu_0 M_{\text{s},\zeta} - (P_{\text{s},r,\text{right}} - P_{\text{s},r,\text{left}})/(\epsilon_0\triangle\xi) - K_{\zeta,\text{s,imp}}$$

$$\downarrow \quad \triangle\xi \to 0$$

$$\triangle E_\xi = -j\omega\mu_0 M_{\text{s},\zeta} - \frac{\partial(P_{\text{s},r}/\epsilon_0)}{\partial\xi} - K_{\zeta,\text{s,imp}}$$

Generalized Sheet Transition Conditions (GSTCs)

$$\triangle E_\xi = -j\omega\mu_0 M_{\mathrm{s},\zeta} - \frac{\partial(P_{\mathrm{s},r}/\epsilon_0)}{\partial\xi} - K_{\zeta,\mathrm{s,imp}}$$

$$\triangle H_\xi = j\omega P_{\mathrm{s},\zeta} - \frac{\partial M_{\mathrm{s},r}}{\partial\xi} + J_{\zeta,\mathrm{s,imp}}\xi^-$$

$$\triangle E_\zeta = j\omega\mu_0 M_{\mathrm{s},\xi} - \frac{\partial(P_{\mathrm{s},r}/\epsilon_0)}{\partial\zeta} + K_{\xi,\mathrm{s,imp}}$$

$$\triangle H_\zeta = -j\omega P_{\mathrm{s},\xi} - \frac{\partial M_{\mathrm{s},r}}{\partial\zeta} - J_{\xi,\mathrm{s,imp}}$$

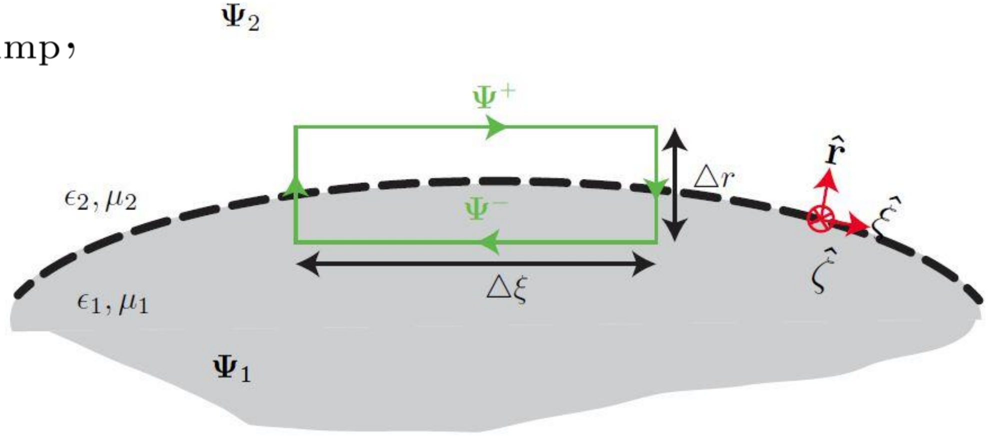
<u>GSTCs</u>:

$$\hat{\mathbf{r}} \times \triangle\mathbf{E} = -j\omega\mu_0\mathbf{M}_{\mathrm{s},\|} + \nabla_\|(P_{\mathrm{s},r}/\epsilon_0) \times \hat{\mathbf{r}} - K_{\|,\mathrm{s,imp}},$$

$$\hat{\mathbf{r}} \times \triangle\mathbf{H} = j\omega\mathbf{P}_{\mathrm{s},\|} - \hat{\mathbf{r}} \times \nabla_\| M_{\mathrm{s},r} + J_{\|,\mathrm{s,imp}},$$

$$\mathbf{P} = \epsilon_0\bar{\bar{\chi}}_{\mathrm{ee}}\mathbf{E}_{\mathrm{av}} + \sqrt{\mu_0\epsilon_0}\bar{\bar{\chi}}_{\mathrm{em}}\mathbf{H}_{\mathrm{av}},$$

$$\mathbf{M} = \sqrt{\epsilon_0/\mu_0}\bar{\bar{\chi}}_{\mathrm{me}}\mathbf{E}_{\mathrm{av}} + \bar{\bar{\chi}}_{\mathrm{mm}}\mathbf{H}_{\mathrm{av}}.$$



Ref 1: Idemen, M. Mithat. Discontinuities in the electromagnetic field. Vol. 40. John Wiley & Sons, 2011.

Ref 2: Achouri, Karim, Mohamed A. Salem, and Christophe Caloz. "General metasurface synthesis based on susceptibility tensors." IEEE Transactions on Antennas and Propagation 63.7 (2015): 2977-2991.

Ref 3: Kuester, Edward F., et al. "Averaged transition conditions for electromagnetic fields at a metafilm." IEEE Transactions on Antennas and Propagation 51.10 (2003): 2641-2651.

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Synthesis Equations

GSTCs:

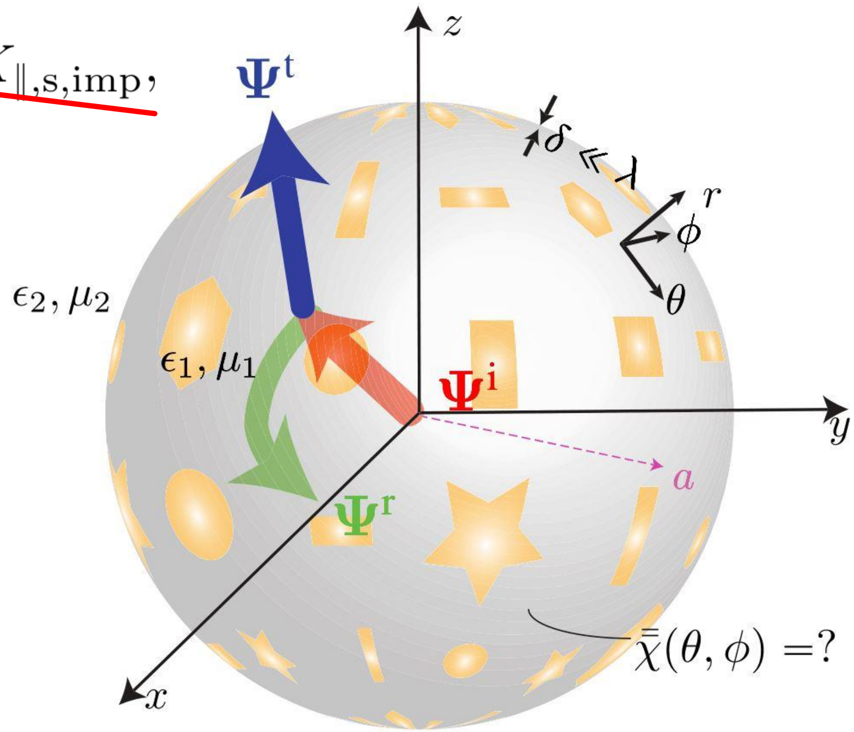
$$\hat{\mathbf{r}} \times \triangle\mathbf{E} = -j\omega\mu_0\mathbf{M}_{\text{s},\parallel} + \nabla_\parallel(P_{\text{s},r}/\epsilon_0) \times \hat{\mathbf{r}} - K_{\parallel,\text{s,imp}},$$

$$\hat{\mathbf{r}} \times \triangle\mathbf{H} = j\omega\mathbf{P}_{\text{s},\parallel} - \hat{\mathbf{r}} \times \nabla_\parallel M_{\text{s},r} + J_{\parallel,\text{s,imp}},$$

$$\mathbf{P} = \epsilon_0\bar{\bar{\chi}}_{\text{ee}}\mathbf{E}_{\text{av}} + \sqrt{\mu_0\epsilon_0}\bar{\bar{\chi}}_{\text{em}}\mathbf{H}_{\text{av}},$$

$$\mathbf{M} = \sqrt{\epsilon_0/\mu_0}\bar{\bar{\chi}}_{\text{me}}\mathbf{E}_{\text{av}} + \bar{\bar{\chi}}_{\text{mm}}\mathbf{H}_{\text{av}}.$$

$$\begin{bmatrix} \chi_{\text{ab}}^{rr}(\theta,\phi) & \chi_{\text{ab}}^{r\theta}(\theta,\phi) & \chi_{\text{ab}}^{r\phi}(\theta,\phi) \\ \chi_{\text{ab}}^{\theta r}(\theta,\phi) & \chi_{\text{ab}}^{\theta\theta}(\theta,\phi) & \chi_{\text{ab}}^{\theta\phi}(\theta,\phi) \\ \chi_{\text{ab}}^{\phi r}(\theta,\phi) & \chi_{\text{ab}}^{\phi\theta}(\theta,\phi) & \chi_{\text{ab}}^{\phi\phi}(\theta,\phi) \end{bmatrix},$$



Assuming normal components are zero:

$$\begin{bmatrix} -\triangle H_\phi \\ \triangle H_\theta \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \chi_{\text{ee}}^{\theta\theta} & \chi_{\text{ee}}^{\theta\phi} \\ \chi_{\text{ee}}^{\phi\theta} & \chi_{\text{ee}}^{\phi\phi} \end{bmatrix} \begin{bmatrix} E_{\theta,\text{av}} \\ E_{\phi,\text{av}} \end{bmatrix} + j\omega\sqrt{\mu_0\epsilon_0} \begin{bmatrix} \chi_{\text{em}}^{\theta\theta} & \chi_{\text{em}}^{\theta\phi} \\ \chi_{\text{em}}^{\phi\theta} & \chi_{\text{em}}^{\phi\phi} \end{bmatrix} \begin{bmatrix} H_{\theta,\text{av}} \\ H_{\phi,\text{av}} \end{bmatrix}$$

$$\begin{bmatrix} \triangle E_\phi \\ -\triangle E_\theta \end{bmatrix} = j\omega\mu_0 \begin{bmatrix} \chi_{\text{mm}}^{\theta\theta} & \chi_{\text{mm}}^{\theta\phi} \\ \chi_{\text{mm}}^{\phi\theta} & \chi_{\text{mm}}^{\phi\phi} \end{bmatrix} \begin{bmatrix} H_{\theta,\text{av}} \\ H_{\phi,\text{av}} \end{bmatrix} + j\omega\sqrt{\mu_0\epsilon_0} \begin{bmatrix} \chi_{\text{me}}^{\theta\theta} & \chi_{\text{me}}^{\theta\phi} \\ \chi_{\text{me}}^{\phi\theta} & \chi_{\text{me}}^{\phi\phi} \end{bmatrix} \begin{bmatrix} E_{\theta,\text{av}} \\ E_{\phi,\text{av}} \end{bmatrix}.$$

Susceptibility Determination (Simplest Case)

Reciprocity(possibly with loss and gain):

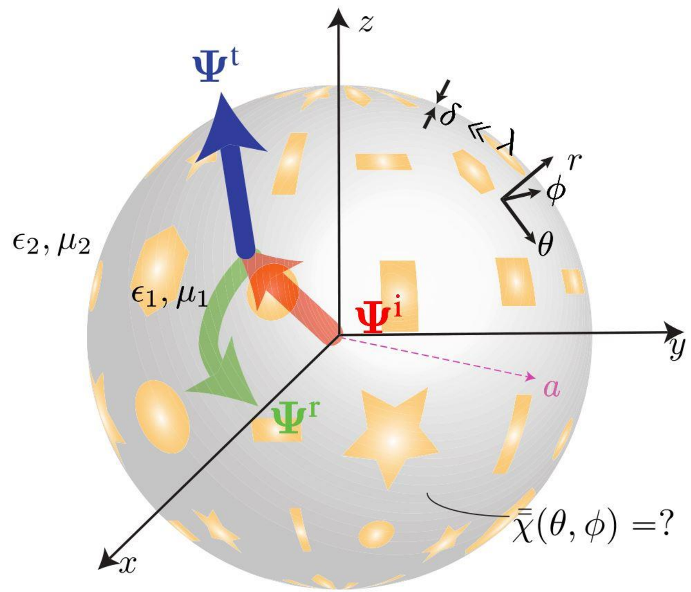
$$\bar{\bar{\chi}}_{\mathrm{ee}}^{T}=\bar{\bar{\chi}}_{\mathrm{ee}},\quad \bar{\bar{\chi}}_{\mathrm{mm}}^{T}=\bar{\bar{\chi}}_{\mathrm{mm}},\quad \bar{\bar{\chi}}_{\mathrm{me}}^{T}=-\bar{\bar{\chi}}_{\mathrm{em}}$$

Loss/gain-less reciprocity:

$$\bar{\bar{\chi}}_{\mathrm{ee}}^{T}=\bar{\bar{\chi}}_{\mathrm{ee}}^{*},\quad \bar{\bar{\chi}}_{\mathrm{mm}}^{T}=\bar{\bar{\chi}}_{\mathrm{mm}}^{*},\quad \bar{\bar{\chi}}_{\mathrm{me}}^{T}=\bar{\bar{\chi}}_{\mathrm{em}}^{*},$$

Non-gyrotropy:

$$\bar{\bar{\chi}}_{\mathrm{ee,mm}}^{\theta\phi}=0,\ \bar{\bar{\chi}}_{\mathrm{ee,mm}}^{\phi\theta}=0,\ \bar{\bar{\chi}}_{\mathrm{em,me}}^{\theta\theta}=0,\ \bar{\bar{\chi}}_{\mathrm{em,me}}^{\phi\phi}=0$$



Non-gyrotropic monoanisotropy (and therefore also reciprocity)

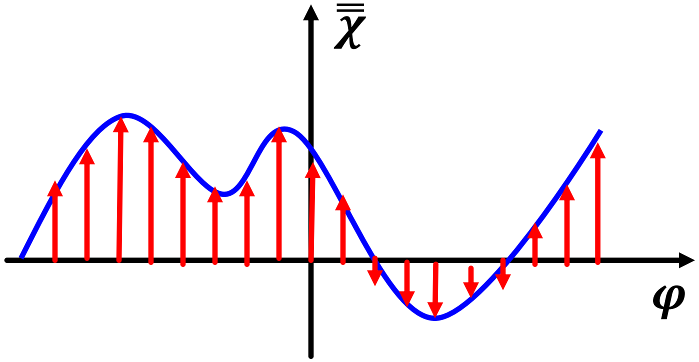
$$\begin{bmatrix}-\triangle H_{\phi}\\ \triangle H_{\theta}\end{bmatrix}=j\omega\epsilon_0\begin{bmatrix}\chi_{\mathrm{ee}}^{\theta\theta} & 0\\ 0 & \chi_{\mathrm{ee}}^{\phi\phi}\end{bmatrix}\begin{bmatrix}E_{\theta,\mathrm{av}}\\ E_{\phi,\mathrm{av}}\end{bmatrix}\qquad \begin{bmatrix}\triangle E_{\phi}\\ -\triangle E_{\theta}\end{bmatrix}=j\omega\mu_0\begin{bmatrix}\chi_{\mathrm{mm}}^{\theta\theta} & 0\\ 0 & \chi_{\mathrm{mm}}^{\phi\phi}\end{bmatrix}\begin{bmatrix}H_{\theta,\mathrm{av}}\\ H_{\phi,\mathrm{av}}\end{bmatrix}$$

<u>Solution</u>:

$$\chi_{\mathrm{ee}}^{\theta\theta}=-\frac{\triangle H_{\phi}}{j\omega\epsilon_0 E_{\theta,\mathrm{av}}}\quad \chi_{\mathrm{mm}}^{\phi\phi}=-\frac{\triangle E_{\theta}}{j\omega\mu_0 H_{\phi,\mathrm{av}}}\quad \chi_{\mathrm{ee}}^{\phi\phi}=\frac{\triangle H_{\theta}}{j\omega\epsilon_0 E_{\phi,\mathrm{av}}}\quad \chi_{\mathrm{mm}}^{\theta\theta}=\frac{\triangle E_{\phi}}{j\omega\mu_0 H_{\theta,\mathrm{av}}}$$

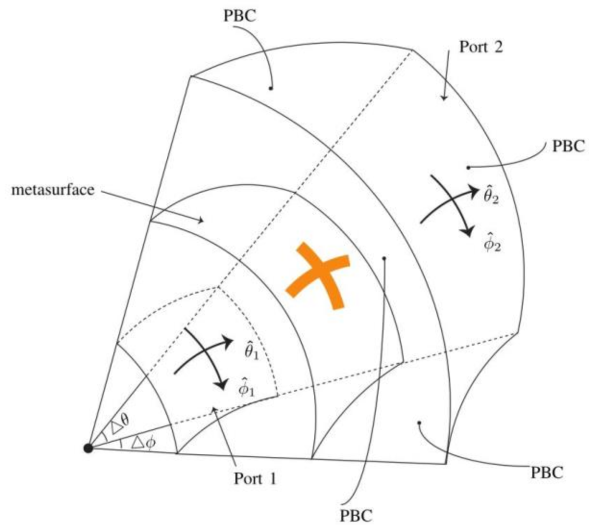
Scattering Parameter Mapping from Susceptibilities

Discretize the susceptibilities



Select a scattering particle

Compute the scattering parameters



Adjust the geometry

No

Extract the susceptibilities

$$\chi_{\text{ee}}^{\theta\theta} = \frac{-2[T_\theta - (1 - R_\theta)]}{j\omega\epsilon_0\eta(T_\theta + 1 + R_\theta)},$$

$$\chi_{\text{ee}}^{\phi\phi} = \frac{2[T_\phi - (1 - R_\phi])}{j\omega\epsilon_0\eta(T_\phi + 1 + R_\phi)},$$

$$\chi_{\text{mm}}^{\theta\theta} = \frac{2\eta[T_\phi - (1 + R_\phi)]}{j\omega\mu_0(T_\phi + 1 - R_\phi)},$$

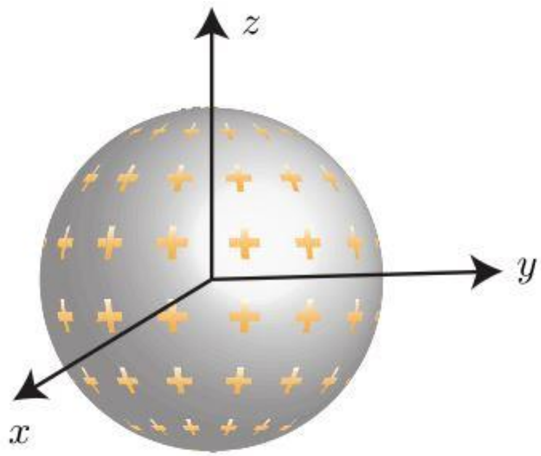
$$\chi_{\text{mm}}^{\phi\phi} = \frac{-2\eta[T_\theta - (1 + R_\theta)]}{j\omega\mu_0(T_\theta + 1 - R_\theta)}.$$

compare

Yes

output

Combine the so designed unit cells



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Illusion Transformation

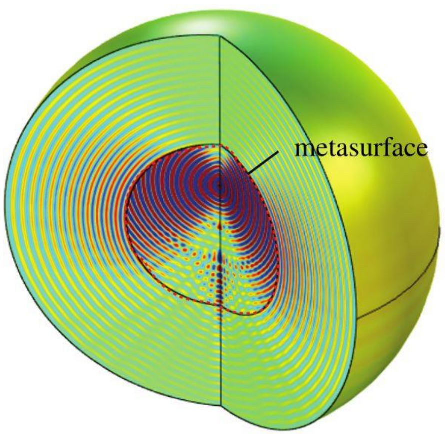
$$E_\theta^{\mathrm{i}}|_{r=a^-} = -\frac{I\ell k_0^2}{j\omega\epsilon_0}\frac{e^{-jk_0R}}{4\pi R}\frac{a^-\sin\theta}{R}\frac{R^2+(a^-)^2-z_0^2}{2Ra^-},$$

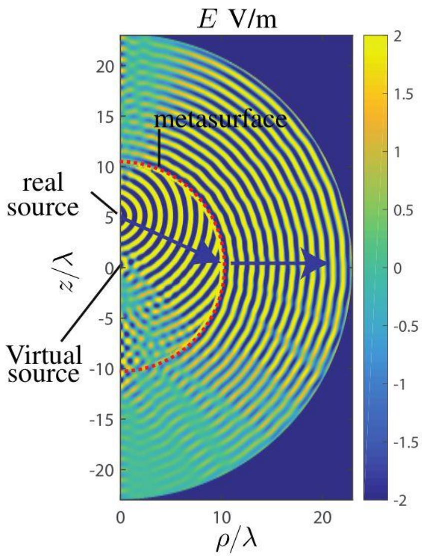
$$H_\phi^{\mathrm{i}}|_{r=a^-} = I\ell jk_0\frac{e^{-jk_0R}}{4\pi R}\frac{a^-\sin\theta}{R}$$

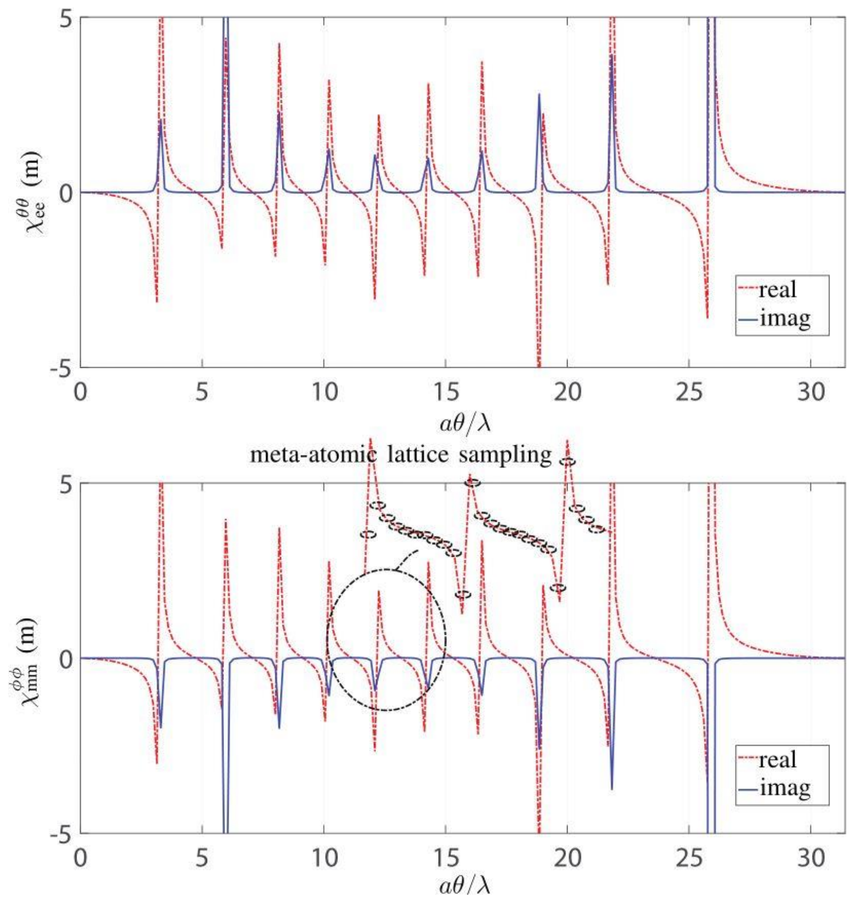
$$E_\theta^{\mathrm{t}}|_{r=a^+} = -T\frac{I\ell k_0^2}{j\omega\epsilon_0}\frac{e^{-jk_0a^+}}{4\pi a^+}\sin\theta,$$

$$H_\phi^{\mathrm{t}}|_{r=a^+} = TI\ell jk_0\frac{e^{-jk_0a^+}}{4\pi a^+}\sin\theta$$

$$\chi_{\mathrm{ee}}^{\theta\theta} = -\frac{\triangle H_\phi}{j\omega\epsilon_0 E_{\theta,\mathrm{av}}}, \quad \chi_{\mathrm{mm}}^{\phi\phi} = -\frac{\triangle E_\theta}{j\omega\mu_0 H_{\phi,\mathrm{av}}}$$



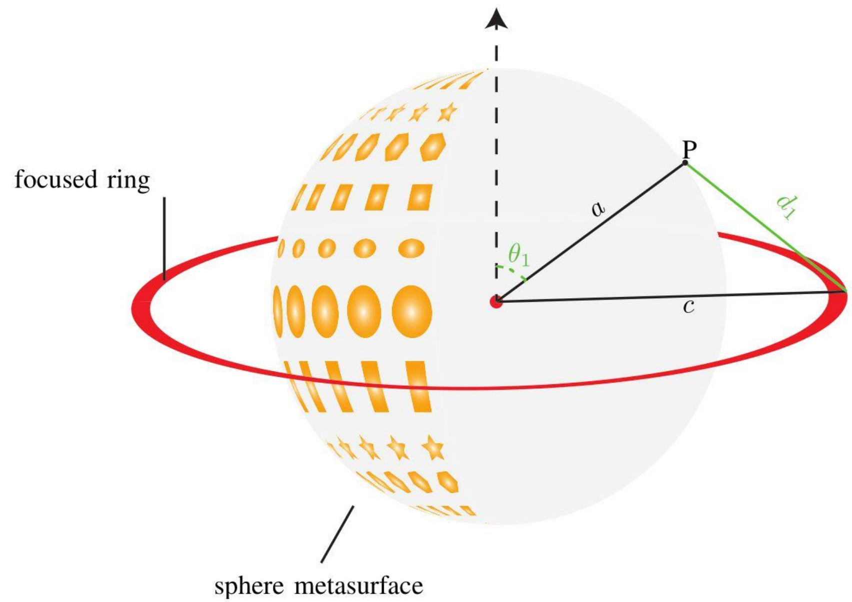




Ring Focusing

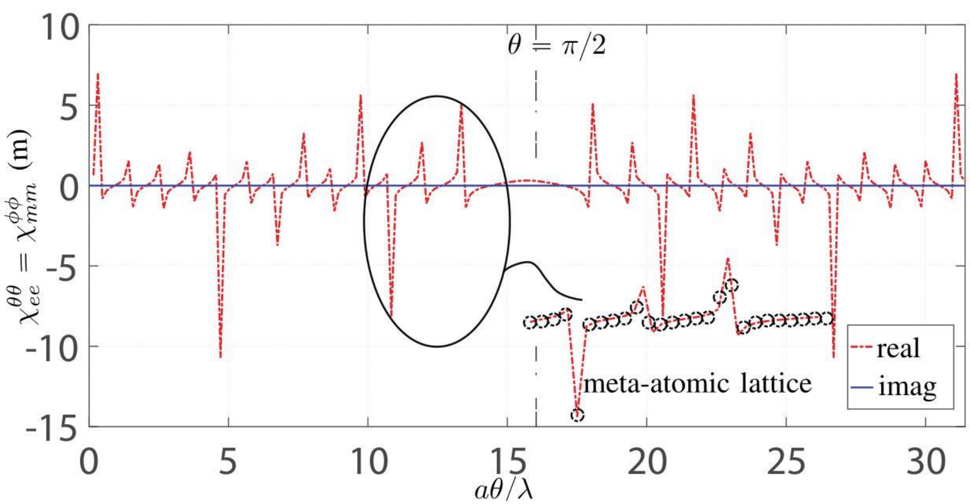
$$E^{\mathrm{i}}_{\theta}|_{r=a^-} = -\frac{I\ell}{j\omega\epsilon_0}\frac{e^{-jk_0a^-}}{4\pi a^-}k_0^2\sin(\theta),$$
$$H^{\mathrm{i}}_{\phi}|_{r=a^-} = I\ell\frac{e^{-jk_0a^-}}{4\pi a^-}jk_0\sin(\theta)$$

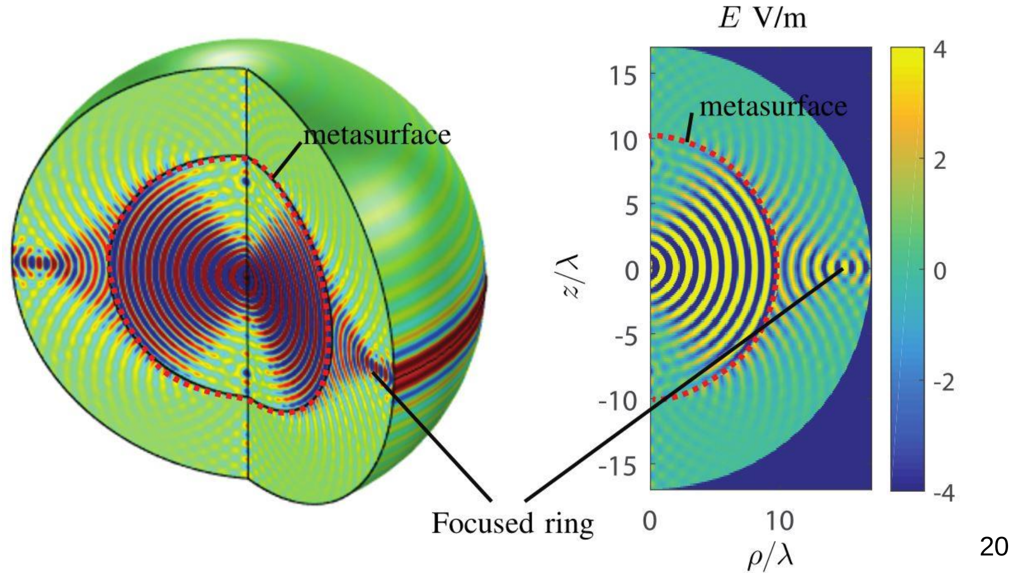
$$E^{\mathrm{t}}_{\theta}|_{r=a^+} = T\eta e^{jk_0d(\theta)},$$
$$H^{\mathrm{t}}_{\phi}|_{r=a^+} = Te^{jk_0d(\theta)}$$



$$-jk_0a + \Phi_P - jk_0d(\theta) = -jk_0[a + d(\theta)] + \Phi_P = \text{const.}$$

$$d(\theta) = \sqrt{a^2 + c^2 - 2ac\sin(\theta)}$$





Birefringence (double transformation)

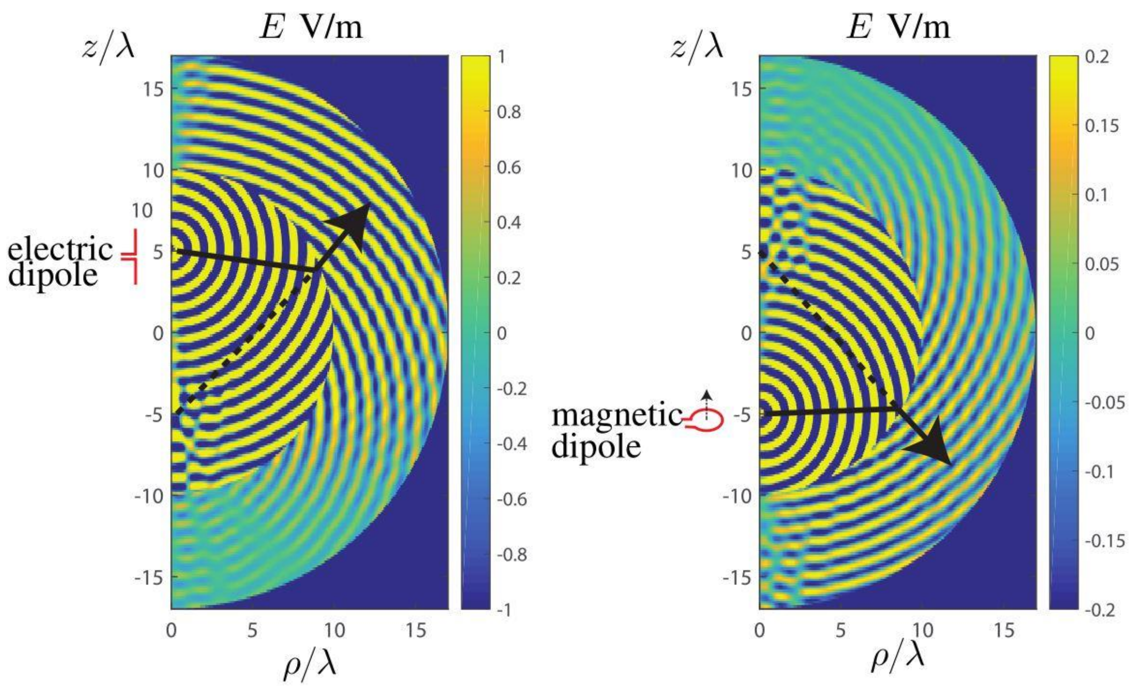
$$E^{\mathrm{i}}_{\theta 1}|_{r=a^-} = I\ell jk_0\eta\frac{e^{-jk_0R_1}}{4\pi R_1}\frac{a^-\sin\theta}{R_1}\frac{R_1^2+(a^-)^2-z_1^2}{2R_1a^-},$$
$$H^{\mathrm{i}}_{\phi 1}|_{r=a^-} = I\ell jk_0\frac{e^{-jk_0R_1}}{4\pi R_1}\frac{a^-\sin\theta}{R_1}$$

$$E^{\mathrm{t}}_{\theta 1}|_{r=a^+} = T_1I\ell jk_0\eta\frac{e^{-jk_0R_2}}{4\pi R_2}\frac{a^+\sin\theta}{R_2}\frac{R_2^2+(a^+)^2-z_2^2}{2R_2a^+},$$
$$H^{\mathrm{t}}_{\phi 1}|_{r=a^+} = T_1I\ell\frac{e^{-jk_0R_2}}{4\pi R_2}jk_0\frac{a^+\sin\theta}{R_2}$$

$$E^{\mathrm{i}}_{\phi 2}|_{r=a^-} = IA\eta k_0^2\frac{e^{-jk_0R_2}}{4\pi R_2}\frac{a^-\sin\theta}{R_2},$$
$$H^{\mathrm{i}}_{\theta 2}|_{r=a^-} = -IAk_0^2\frac{e^{-jk_0R_2}}{4\pi R_2}\frac{a^-\sin\theta}{R_2}\frac{R_2^2+(a^-)^2-z_2^2}{2R_2a^-}$$

$$E^{\mathrm{t}}_{\phi 2}|_{r=a^+} = T_2IA\eta k_0^2\frac{e^{-jk_0R_1}}{4\pi R_1}\frac{a^+\sin\theta}{R_1},$$
$$H^{\mathrm{t}}_{\theta 2}|_{r=a^+} = -T_2IAk_0^2\frac{e^{-jk_0R_1}}{4\pi R_1}\frac{a^+\sin\theta}{R_1}\frac{R_1^2+(a^+)^2-z_1^2}{2R_1a^+}$$

$$\chi_{\mathrm{ee}}^{\theta\theta} = \frac{1}{j\omega\epsilon_0}\frac{\triangle H_{\phi 2}E_{\phi 1,\mathrm{av}}-\triangle H_{\phi 1}E_{\phi 2,\mathrm{av}}}{E_{\theta 1,\mathrm{av}}E_{\phi 2,\mathrm{av}}-E_{\theta 2,\mathrm{av}}E_{\phi 1,\mathrm{av}}},$$
$$\chi_{\mathrm{ee}}^{\theta\phi} = \frac{1}{j\omega\epsilon_0}\frac{\triangle H_{\phi 2}E_{\theta 1,\mathrm{av}}-\triangle H_{\phi 1}E_{\theta 2,\mathrm{av}}}{E_{\phi 1,\mathrm{av}}E_{\theta 2,\mathrm{av}}-E_{\phi 2,\mathrm{av}}E_{\theta 1,\mathrm{av}}},$$
$$\chi_{\mathrm{ee}}^{\phi\theta} = \frac{1}{j\omega\epsilon_0}\frac{\triangle H_{\theta 1}E_{\phi 2,\mathrm{av}}-\triangle H_{\theta 2}E_{\phi 1,\mathrm{av}}}{E_{\theta 1,\mathrm{av}}E_{\phi 2,\mathrm{av}}-E_{\theta 2,\mathrm{av}}E_{\phi 1,\mathrm{av}}},$$
$$\chi_{\mathrm{ee}}^{\phi\phi} = \frac{1}{j\omega\epsilon_0}\frac{\triangle H_{\theta 1}E_{\theta 2,\mathrm{av}}-\triangle H_{\theta 2}E_{\theta 1,\mathrm{av}}}{E_{\phi 1,\mathrm{av}}E_{\theta 2,\mathrm{av}}-E_{\phi 2,\mathrm{av}}E_{\theta 1,\mathrm{av}}},$$
$$\chi_{\mathrm{mm}}^{\theta\theta} = \frac{1}{j\omega\mu_0}\frac{\triangle E_{\phi 1}H_{\phi 2,\mathrm{av}}-\triangle E_{\phi 2}H_{\phi 1,\mathrm{av}}}{H_{\theta 1,\mathrm{av}}H_{\phi 2,\mathrm{av}}-H_{\theta 2,\mathrm{av}}H_{\phi 1,\mathrm{av}}},$$
$$\chi_{\mathrm{mm}}^{\theta\phi} = \frac{1}{j\omega\mu_0}\frac{\triangle E_{\phi 1}H_{\theta 2,\mathrm{av}}-\triangle E_{\phi 2}H_{\theta 1,\mathrm{av}}}{H_{\phi 1,\mathrm{av}}H_{\theta 2,\mathrm{av}}-H_{\phi 2,\mathrm{av}}H_{\theta 1,\mathrm{av}}},$$
$$\chi_{\mathrm{mm}}^{\phi\theta} = \frac{1}{j\omega\mu_0}\frac{\triangle E_{\theta 2}H_{\phi 1,\mathrm{av}}-\triangle E_{\theta 1}H_{\phi 2,\mathrm{av}}}{H_{\theta 1,\mathrm{av}}H_{\phi 2,\mathrm{av}}-H_{\theta 2,\mathrm{av}}H_{\phi 1,\mathrm{av}}},$$
$$\chi_{\mathrm{mm}}^{\phi\phi} = \frac{1}{j\omega\mu_0}\frac{\triangle E_{\theta 2}H_{\theta 1,\mathrm{av}}-\triangle E_{\theta 1}H_{\theta 2,\mathrm{av}}}{H_{\phi 1,\mathrm{av}}H_{\theta 2,\mathrm{av}}-H_{\phi 2,\mathrm{av}}H_{\theta 1,\mathrm{av}}}$$



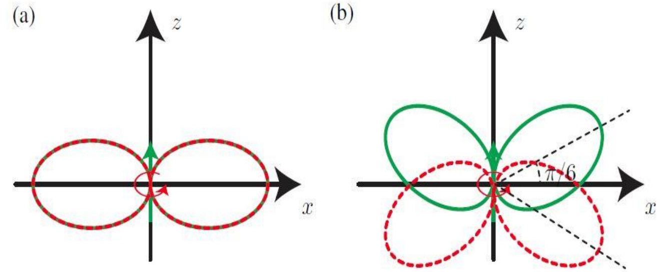
Multiple Beam Forming

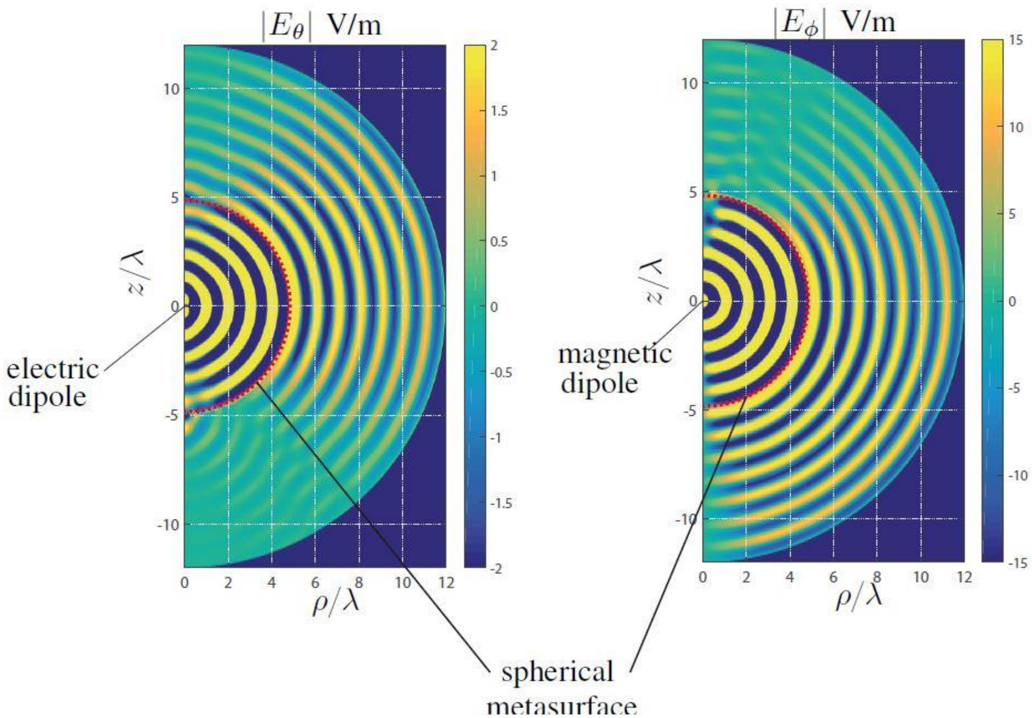
$$E_\theta^{\mathrm{i}}|_{r=a^-} = \frac{j\eta I}{2\pi R} e^{-jk_0 R} \frac{\cos(k_0\ell/2\cos(\theta)) - \cos(k_0\ell/2)}{\sin(\theta}),$$

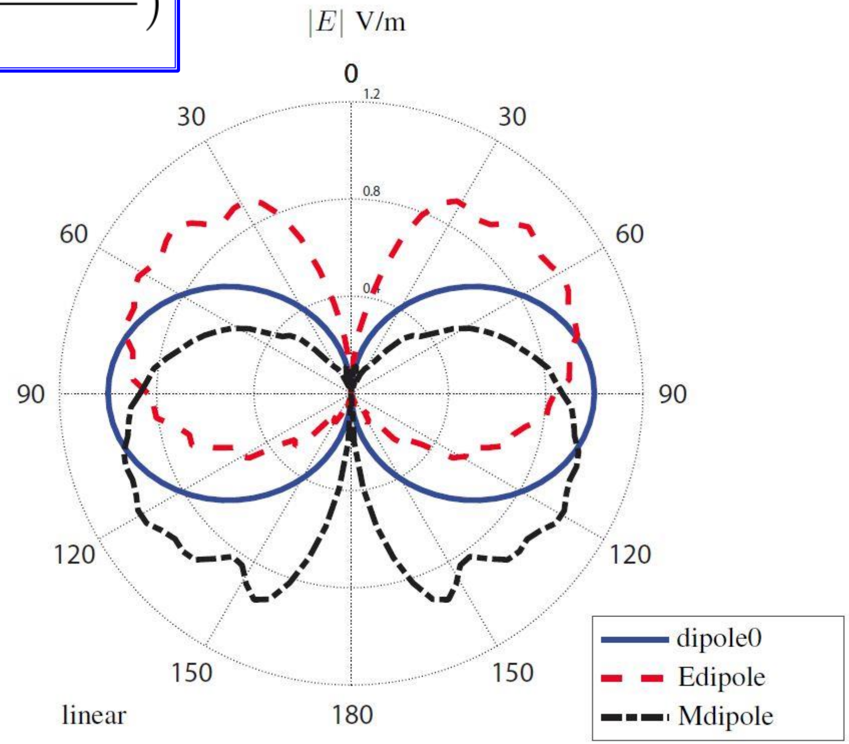
$$H_\phi^{\mathrm{i}}|_{r=a^-} = \frac{jI}{2\pi R} e^{-jk_0 R} \frac{\cos(k_0\ell/2\cos(\theta)) - \cos(k_0\ell/2)}{\sin(\theta})$$

$$E_\theta^{\mathrm{t}}|_{r=a^+} = \frac{j\eta I}{2\pi R} e^{-jk_0 R} \frac{\cos(k_0\ell/2\cos(\theta_{\mathrm{c}})) - \cos(k_0\ell/2)}{\sin(\theta_{\mathrm{c}}}),$$

$$H_\phi^{\mathrm{t}}|_{r=a^+} = \frac{jI}{2\pi R} e^{-jk_0 R} \frac{\cos(k_0\ell/2\cos(\theta_{\mathrm{c}})) - \cos(k_0\ell/2)}{\sin(\theta_{\mathrm{c}}})$$







Conclusion

- **Extension of susceptibility-GSTC synthesis to spherical metasurfaces**

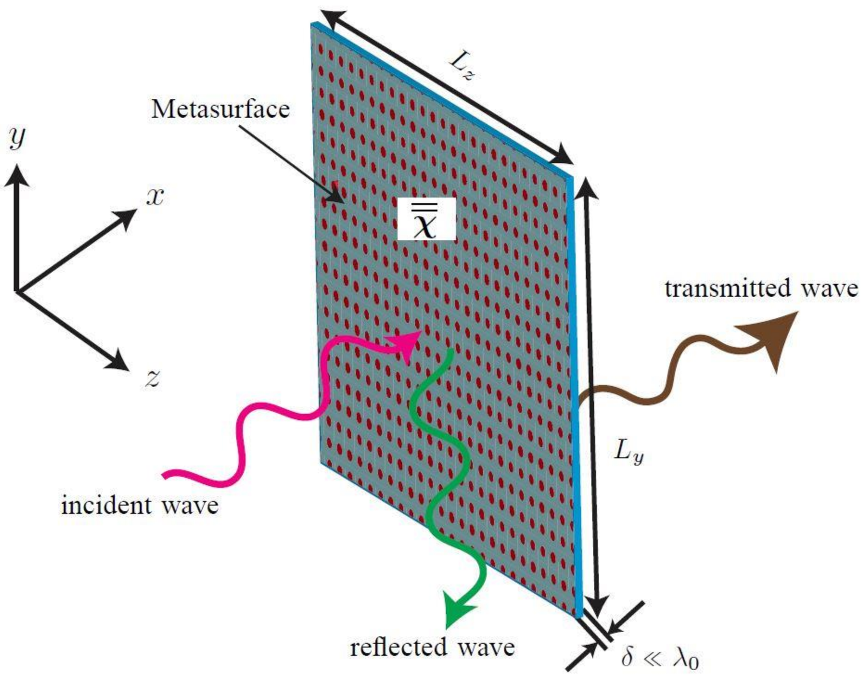
- **Future works:**

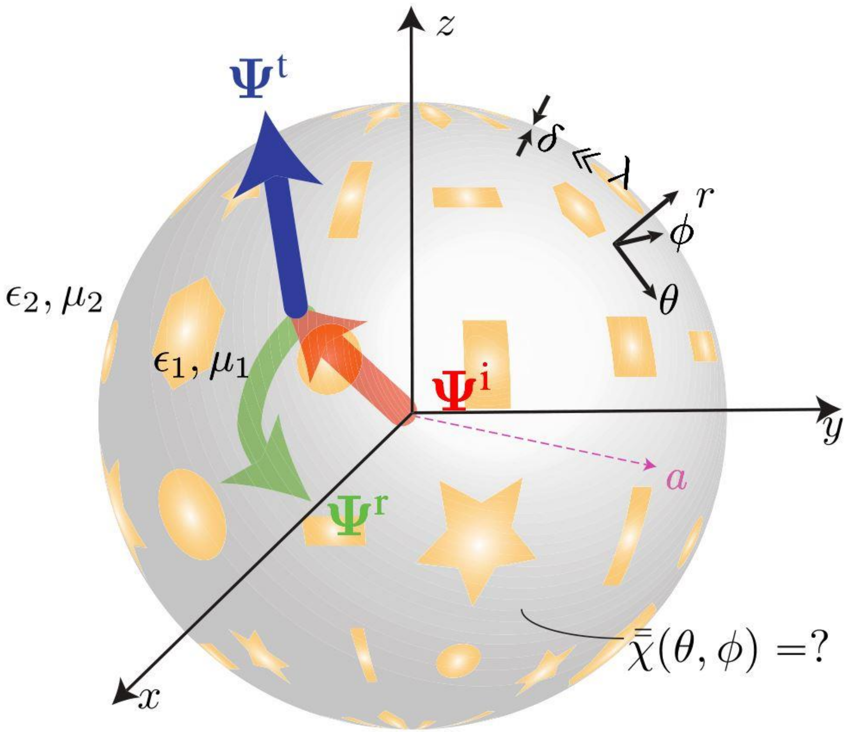
❖ Porous cavity (multiple internal scattering) problem

❖ Scattering particle design (size reduction from equator to poles)

❖ Fabrication & testing

❖ Application to beam forming





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